Linear Programming Solution Special Cases

(1) Some LP problems have an infinite number of optimal solutions (*alternative or multiple optimal solutions*)

Max $Z = 3x1 + 2x2$
s.t.
(1) $1/40\ x1 + 1/60\ x2 \le 1$ þ $120/40\ x1 + 120/60\ x2 \le 120$
(2) $1/50\ x1 + 1/50\ x2 \le 1$ $3x1 + 2x2 \le 120$
and $x1, x2 \ge 0$

Solution:

Z = 120	x1 = 40	x2 = 0	(corner point feasible solution)
Z = 120	x1 = 20	x2 = 30	(corner point feasible solution)
Z = 120	x1 = 30	x2 = 15	(point between cpf solutions)

(2) Some LP problems have no feasible solutions (*infeasible solutions*)

Max $Z = 3x1 + 2x2$
s.t.
(1) $1/40\ x1 + 1/60\ x2 \le 1$ þ $30/40 + 20/60 = 1.083$ (constraint violated)
(2) $1/50\ x1 + 1/50\ x2 \le 1$ þ $30/50 + 20/50 = 1.00$ (constraint not violated)
(3) $x1 \ge 30$
(4) $x2 \ge 20$
and $x1, x2 \ge 0$

(3) Some LP problems have points in the feasible region with arbitrarily large (in a max problem) *Z*-values (*unbounded solutions*)

Max $Z = 3x1 - x2$
s.t.
(1) $x1 - x2 \le 1$ þ Neither constraint (1) or (2) places
(2) $2\ x1 + x2 \ge 6$ þ upper limits on values of x1 and x2
and $x1, x2 \ge 0$

Four Possible Solution Cases

Case 1 The LP has a *unique optimal solution*.

Case 2 The LP has *alternative* or *multiple optimal solutions*: Two or more extreme points are optimal, and the LP will have an infinite number of optimal solutions.

Case 3 The LP is *infeasible*: The feasible region contains no points.

Case 4 The LP is *unbounded*: There are points in the feasible region with arbitrarily large *Z*-values (max problem) or arbitrarily small *Z*-values (min problem).