<u>Goal</u> <u>Programming</u> - "Bank Investment Example"

Every investor must tradeoff return versus risk in deciding how to allocate his or her available funds. The opportunities that promise the greatest profits are almost always the ones that present the most serious risks. Commercial banks must be especially careful in balancing return and risk because legal and ethical obligations demand that they avoid undo hazards, yet their goal as a business enterprise is to maximize profit. This dilemma leads naturally to multiobjective optimization of investment that includes both profit and risk criteria.

This investment example adopts this multiobjective approach to a fictitious Bank Three. Bank Three has a modest $20 million capital, with $150 million in demand deposits (checking accounts) and $80 million in time deposits (savings accounts and certificates of deposit). The table below displays the categories among which the bank must divide its capital and deposited funds. Rates of return are also provided for each category together with other information related to risk.

Bank Three Investment Opportunities				
Investment Category (*j*)	Return Rate (%)	Liquid Part (%)	Required Capital (%)	Risk Asset? (%)
1. Cash	0.0	100.0	0.0	No
2. Short Term	4.0	99.5	0.5	No
3. Government: 1 to 5 years	4.5	96.0	4.0	No
4. Government: 5 to 10 years	5.5	90.0	5.0	No
5. Government: over 10 years	7.0	85.0	7.5	No
6. Installment loans	10.5	0.0	10.0	Yes
7. Mortgage loans	8.5	0.0	10.0	Yes
8. Commercial loans	9.2	0.0	10.0	Yes

The first goal of any private business is to maximize profit. Using rates of return from the table above, this produces the objective function

$$\text{Max } P = 0.040x2 + 0.045x3 + 0.055x4 + 0.070x5 + 0.105x6 + 0.085x7 + 0.092x8 \qquad \text{[profit]}$$

It is less clear how to quantify investment risk. Two commonly used ratio measures are employed.

One is the *capital-adequacy ratio*, expressed as the ratio of required capital for bank solvency to actual capital. A low value indicates minimum risk. The "required capital" rates in the table approximate U.S. government formulas used to compute this ratio, and Bank Three's present capital is $20 million. Thus the second objective can be expressed as

$$\text{Min } C = 1/20\,(0.005x2 + 0.0405x3 + 0.050x4 + 0.075x5 + 0.100x6 + 0.100x7 + 0.100x8) \qquad \text{[capital-adequacy]}$$

Another measure of risk focuses on illiquid *risk assets*. A low risk asset/capital ratio indicates a financially secure institution. For this example, this third measure of success is expressed as

$$\text{Min } R = 1/20\,(x6 + x7 + x8) \qquad \text{[risk-asset]}$$

To complete a model of Bank Three's investment plans, we must describe the relevant constraints. This example will assume five types:

1. Investments must sum to the available capital and deposit funds.
2. Cash reserves must be at least 14% of demand deposits plus 4% of time deposits.
3. The portion of investments considered liquid should be at least 47% of demand deposits plus 36% of time deposits.
4. At least 5% of funds should be invested in each of the eight categories, for diversity.
5. At least 30% of funds should be invested in commercial loans, to maintain the bank's community status.

Combining the three objective functions above with these five systems of constraints completes a multiobjective linear programming model of Bank Three's investment problem:

$$\text{Max } P = 0.040x2 + 0.045x3 + 0.055x4 + 0.070x5 + 0.105x6 + 0.085x7 + 0.092x8 \qquad \text{[profit]}$$

$$\text{Min } C = 1/20\,(0.005x2 + 0.0405x3 + 0.050x4 + 0.075x5 + 0.100x6 + 0.100x7 + 0.100x8) \qquad \text{[capital-adequacy]}$$

$$\text{Min } R = 1/20\,(x6 + x7 + x8) \qquad \text{[risk-asset]}$$

s.t.

(1) $x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 = (20 + 150 + 80)$ [invest all]

(2) $x1 \geq 0.14\,(150) + 0.04\,(80)$ [cash reserve]

(3) $1.00x1 + 0.995x2 + 0.960x3 + 0.900x4 + 0.850x5 \geq 0.47\,(150) + 0.36\,(80)$ [liquidity]

(4) $x_j \geq 0.05\,(20 + 150 + 80)$ for all $j = 1, \ldots 8$ [diversification]

(5) $x8 \geq 0.30\,(20 + 150 + 80)$ [commercial]

Here solutions are evaluated on three criteria: profit, capital-adequacy ratio, and risk-asset ratio. Assume that instead of seeking ever higher levels of the first criterion and lower values of the last two, we set some goals:

profit $\geq$ 18.5
capital-adequacy ratio $\leq$ 0.8
risk-asset ratio $\leq$ 7.0

The three objective functions can then be stated as goals in the following manner:

Goal 1 $0.040x2 + 0.045x3 + 0.055x4 + 0.070x5 + 0.105x6 + 0.085x7 + 0.092x8 \geq 18.5$ [profit]

Goal 2 $1/20\,(0.005x2 + 0.0405x3 + 0.050x4 + 0.075x5 + 0.100x6 + 0.100x7 + 0.100x8) \leq 0.8$ [capital-adequacy]

Goal 3 $1/20\,(x6 + x7 + x8) \leq 7.0$ [risk-asset]

The above goals may be thought of as *soft constraints*. Soft constraints such as the criteria targets of goal programming specify requirements that are desirable to satisfy but which may still be violated in feasible solutions. Once target levels have been specified for soft constraints, we proceed to a more familiar mathematical programming formulation by adding constraints that enforce goal achievement. However, we cannot just impose the constraint that each objective meet its goal. There may be no solution that simultaneously achieves the desired levels of all soft constraints. Instead we introduce new *deficiency variables.* Nonnegative deficiency variables are introduced to model the extent of violation in goal or other soft constraints that need not be rigidly enforced. With a $\geq$ target, the deficiency is the under achievement. With a $\leq$ target, it is the excess. With = soft constraints, deficiency variables are included for both under- and over achievement.

In the three-objective Bank Three example, we enforce goal levels with deficiency variables:

$d1^-$ = amount profit falls short of its goal
$d2^+$ = amount capital-adequacy ratio exceeds its goal
$d3^+$ = amount risk-asset ratio exceeds its goal

Using equal goal weights, the Bank Three problem produces the following linear goal programming model:

Min $d1^- + d2^+ + d3^+$
s.t.

$0.040x2 + 0.045x3 + 0.055x4 + 0.070x5 + 0.105x6 + 0.085x7 + 0.092x8 + d1^- \geq 18.5$ [profit]

$1/20\,(0.005x2 + 0.0405x3 + 0.050x4 + 0.075x5 + 0.100x6 + 0.100x7 + 0.100x8) - d2^+ \leq 0.8$ [capital-adequacy]

$1/20\,(x6 + x7 + x8) - d3^+ \leq 7.0$ [risk-asset]

$x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 = (20 + 150 + 80)$ [invest all]

$x1 \geq 0.14\,(150) + 0.04\,(80)$ [cash reserve]

$1.00x1 + 0.995x2 + 0.960x3 + 0.900x4 + 0.850x5 \geq 0.47\,(150) + 0.36\,(80)$ [liquidity]

$x_j \geq 0.05\,(20 + 150 + 80)$ for all $j = 1, \ldots 8$ [diversification]

$x8 \geq 0.30\,(20 + 150 + 80)$ [commercial]

$x1, \ldots x8 \geq 0$ $\quad d1, d2, d3 \geq 0$

Goal Programming Solution of the Bank Three Example					
	(1) Equal Weights	(2) Unequal Weights	(3) Preempt Profit	(4) Preempt Profit,CA	(5) Preempt One Step
Profit goal weight	1	1	1	0	10,000
Cap.-adequacy goal weight	1	10	0	1	100
Risk-asset goal weight	1	1	0	0	1
Extra constraints	-	-	-	d1=0	-
Profit	18.50	17.53	18.50	18.50	18.50
Deficiency, *d1*	0.00	0.97	0.00	0.00	0.00
Capital-adequacy ratio	0.928	0.815	0.943	0.919	0.919
Deficiency, *d2*	0.128	0.015	0.143	0.119	0.119
Risk-asset ratio	7.000	7.000	7.097	7.158	7.158
Deficiency, *d3*	0.000	0.000	0.097	0.158	0.158
Cash, *x1*	24.20	24.20	24.20	24.20	24.20
Short term, *x2*	16.03	48.30	12.50	19.73	19.73
Government, 1-5, *x3*	12.50	12.50	12.50	12.50	12.50
Government, 5-10, *x4*	12.50	12.50	12.50	12.50	12.50
Government, over 10, *x5*	44.77	12.50	46.37	37.91	37.91
Installment, *x6*	52.50	52.50	41.08	55.67	55.67
Mortgages, *x7*	12.50	12.50	12.50	12.50	12.50
Commercial, *x8*	75.00	75.00	88.36	75.00	75.00

Column (1) shows an optimal solution to an equal-weighted linear goal programming problem. Notice how it seeks only the $18.5 million goal for profit and the 7.0 goal for risk-asset ratio. Once corresponding deficiency variables d1 and d3 are driven to 0.0, effort can be directed toward the remaining capital-adequacy goal. Column (2) shows the effect of multiplying the capital adequacy weight by 10. Now the profit slips below its $18.5 million goal (to $17.53 million), but the capital-adequacy ratio deficiency is reduced. Columns (3) and (4) illustrate the sequential preemptive variant of goal programming. Column (5) yields the same solution by weighting the objective function.