

Modeling Inventory Problems as Transportation Problems

	1	2	3	4	Dummy	Supply
Initial	0	20	40	60	0	10
Qtr 1 RT	400	420	440	460	0	40
Qtr 1 OT	450	470	490	510	0	150
Qtr 2 RT	M	400	420	440	0	40
Qtr 2 OT	M	450	470	490	0	150
Qtr 3 RT	M	M	400	420	0	40
Qtr 3 OT	M	M	450	470	0	150
Qtr 4 RT	M	M	M	400	0	40
Qtr 4 OT	M	M	M	450	0	150
Demand	40	60	75	25	570	

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(1) Standard LP Format:

$$\begin{aligned} \text{Minimize } Z = & 20x_{12} + 40x_{13} + 60x_{14} + 400x_{21} + 420x_{22} + 440x_{23} + 460x_{24} + 450x_{31} \\ & + 470x_{32} + 490x_{33} + 510x_{34} + 100000x_{41} + 400x_{42} + 420x_{43} + 440x_{44} \\ & + 100000x_{51} + 450x_{52} + 470x_{53} + 490x_{54} + 100000x_{61} + 100000x_2 \\ & + 400x_{63} + 420x_{64} + 100000x_{71} + 100000x_{72} + 450x_{73} + 470x_{74} \\ & + 100000x_{81} + 100000x_{82} + 100000x_{83} + 400x_{84} + 100000x_{91} \\ & + 100000x_{92} + 100000x_{93} + 450x_{94} \end{aligned}$$

s.t.

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15}$	= 10	S1
$x_{21} + x_{22} + x_{23} + x_{24} + x_{25}$	= 40	S2
$x_{31} + x_{32} + x_{33} + x_{34} + x_{35}$	= 150	S3
$x_{41} + x_{42} + x_{43} + x_{44} + x_{45}$	= 40	S4
$x_{51} + x_{52} + x_{53} + x_{54} + x_{55}$	= 150	S5
$x_{61} + x_{62} + x_{63} + x_{64} + x_{65}$	= 40	S6
$x_{71} + x_{72} + x_{73} + x_{74} + x_{75}$	= 150	S7
$x_{81} + x_{82} + x_{83} + x_{84} + x_{85}$	= 40	S8
$x_{91} + x_{92} + x_{93} + x_{94} + x_{95}$	= 150	S9
$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{81} + x_{91}$	= 40	D1
$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} + x_{72} + x_{82} + x_{92}$	= 60	D2
$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} + x_{73} + x_{83} + x_{93}$	= 75	D3
$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} + x_{74} + x_{84} + x_{94}$	= 25	D4
$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} + x_{75} + x_{85} + x_{95}$	= 570	D5

and

$$x_{ij} \geq 0$$

Optimal Solution: Obj. Func. = 78,450

$x_{11} = 10$	$x_{63} = 40$
$x_{21} = 30$	$x_{73} = 35$
$x_{22} = 10$	$x_{75} = 115$
$x_{35} = 150$	$x_{84} = 25$
$x_{42} = 40$	$x_{85} = 15$
$x_{52} = 10$	$x_{95} = 150$
$x_{55} = 140$	

Modeling Inventory Problems as Transportation Problems

(2) Standard LP Format:

$$\begin{aligned} \text{Minimize } Z = & 20x_{12} + 40x_{13} + 60x_{14} + 400x_{21} + 420x_{22} + 440x_{23} + 460x_{24} + 450x_{31} \\ & + 470x_{32} + 490x_{33} + 510x_{34} + 400x_{42} + 420x_{43} + 440x_{44} \\ & + 450x_{52} + 470x_{53} + 490x_{54} + 400x_{63} + 420x_{64} + 450x_{73} + 470x_{74} \\ & + 400x_{84} + 450x_{94} \end{aligned}$$

s.t.

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} & = 10 & S1 \\ x_{21} + x_{22} + x_{23} + x_{24} + x_{25} & = 40 & S2 \\ x_{31} + x_{32} + x_{33} + x_{34} + x_{35} & = 150 & S3 \\ x_{42} + x_{43} + x_{44} + x_{45} & = 40 & S4 \\ x_{52} + x_{53} + x_{54} + x_{55} & = 150 & S5 \\ x_{63} + x_{64} + x_{65} & = 40 & S6 \\ x_{73} + x_{74} + x_{75} & = 150 & S7 \\ x_{84} + x_{85} & = 40 & S8 \\ x_{94} + x_{95} & = 150 & S9 \end{aligned}$$

$$\begin{aligned} x_{11} + x_{21} + x_{31} & = 40 & D1 \\ x_{12} + x_{22} + x_{32} + x_{42} + x_{52} & = 60 & D2 \\ x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} + x_{73} & = 75 & D3 \\ x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} + x_{74} + x_{84} + x_{94} & = 25 & D4 \\ x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} + x_{75} + x_{85} + x_{95} & = 570 & D5 \end{aligned}$$

and

$$x_{ij} \geq 0$$

Optimal Solution: Obj. Func. = 78,450

$$\begin{aligned} x_{11} &= 10 & x_{63} &= 40 \\ x_{21} &= 30 & x_{73} &= 35 \\ x_{22} &= 10 & x_{75} &= 115 \\ x_{35} &= 150 & x_{84} &= 25 \\ x_{42} &= 40 & x_{85} &= 15 \\ x_{52} &= 10 & x_{95} &= 150 \\ x_{55} &= 140 & & \end{aligned}$$

Modeling Inventory Problems as Transportation Problems

(3) Standard LP Format:

$$\begin{aligned} \text{Minimize } Z = & 20x_{12} + 40x_{13} + 60x_{14} + 400x_{21} + 420x_{22} + 440x_{23} + 460x_{24} + 450x_{31} \\ & + 470x_{32} + 490x_{33} + 510x_{34} + 400x_{42} + 420x_{43} + 440x_{44} \\ & + 450x_{52} + 470x_{53} + 490x_{54} + 400x_{63} + 420x_{64} + 450x_{73} + 470x_{74} \\ & + 400x_{84} + 450x_{94} \end{aligned}$$

s.t.

$$\begin{array}{rcll} x_{11} + x_{12} + x_{13} + x_{14} & \leq & 10 & S1 \\ x_{21} + x_{22} + x_{23} + x_{24} & \leq & 40 & S2 \\ x_{31} + x_{32} + x_{33} + x_{34} & \leq & 150 & S3 \\ x_{42} + x_{43} + x_{44} & \leq & 40 & S4 \\ x_{52} + x_{53} + x_{54} & \leq & 150 & S5 \\ x_{63} + x_{64} & \leq & 40 & S6 \\ x_{73} + x_{74} & \leq & 150 & S7 \\ x_{84} & \leq & 40 & S8 \\ x_{94} & \leq & 150 & S9 \\ \\ x_{11} + x_{21} + x_{31} & = & 40 & D1 \\ x_{12} + x_{22} + x_{32} + x_{42} + x_{52} & = & 60 & D2 \\ x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} + x_{73} & = & 75 & D3 \\ x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} + x_{74} + x_{84} + x_{94} & = & 25 & D4 \end{array}$$

and

$$x_{ij} \geq 0$$

Optimal Solution:

$$\text{Obj. Func.} = 78,450$$

$$x_{11} = 10 \quad x_{63} = 40$$

$$x_{21} = 30 \quad x_{73} = 35$$

$$x_{22} = 10 \quad S_7 = 115$$

$$S_3 = 150 \quad x_{84} = 25$$

$$x_{42} = 40 \quad S_8 = 15$$

$$x_{52} = 10 \quad S_9 = 150$$

$$S_5 = 140$$

where S_3, S_5, S_7, S_8, S_9
are slack variables

Modeling Inventory Problems as Transportation Problems

```

DATA NEW4;
INPUT D1 D2 D3 D4 COMPUTER QUARTERS$;
CARDS;
    40    60    75    25    .    .
    0    20    40    60    10    S1
    400   420   440   460   40    S2
    450   470   490   510   150   S3
10000   400   420   440   40    S4
10000   450   470   490   150   S5
10000  10000   400   420   40    S6
10000  10000   450   470   150   S7
10000  10000  10000   400   40    S8
10000  10000  10000   450   150   S9
;
PROC TRANS NOTHRUNET;
    TAILNODE QUARTERS;
    HEADNODE D1-D4;
    SUPPLY    COMPUTER;
    TITLE1 'INVENTORY PROBLEM AS TRANSPORTATION PROBLEM';
RUN;
PROC PRINT;
RUN;

```

INVENTORY PROBLEM AS TRANSPORTATION PROBLEM

OBS	QUARTERS	COMPUTER	D1	D2	D3	D4	_DUAL_
1	_DEMAND_	.	40	60	75	25	.
2	S1	10	0	10	0	0	- 430
3	S2	40	40	0	0	0	- 30
4	S3	150	0	0	0	0	0
5	S4	40	0	40	0	0	- 50
6	S5	150	0	10	0	0	0
7	S6	40	0	0	40	0	- 50
8	S7	150	0	0	35	0	0
9	S8	40	0	0	0	25	0
10	S9	150	0	0	0	0	0
11	_DUAL_	.	430	450	450	400	.

NOTE: Optimum solution total = 78450 .