

**Table 8.31 Cost and Requirements Table for the P&T Co. Transshipment Problem
Formulated as a Transportation Problem**

		"Destination"												Supply
		Cannery			Junction					Warehouse				
		1	2	3	4	5	6	7	8	9	10	11	12	
"Source"	1	0	146	M	324	286	M	M	M	452	505	M	871	375
Cany.	2	146	0	M	373	212	570	609	M	335	407	688	784	425
	3	M	M	0	658	M	405	419	158	M	685	359	673	400
	4	322	371	656	0	262	398	430	M	503	234	329	M	300
	5	284	210	M	262	0	406	421	644	305	207	464	558	300
Junc.	6	M	569	403	398	406	0	81	272	597	253	171	282	300
	7	M	608	418	431	422	81	0	287	613	280	236	229	300
	8	M	M	158	M	647	274	288	0	831	501	293	482	300
	9	453	336	M	505	307	599	615	831	0	359	706	587	300
Wrhs.	10	505	407	683	235	208	254	281	500	357	0	362	341	300
	11	M	687	357	329	464	171	236	290	705	362	0	457	300
	12	868	781	670	M	558	282	229	480	587	340	457	0	300
Demand		300	300	300	300	300	300	300	300	380	365	370	385	

The P&T Co. Transshipment Problem

LP Format 1: Transshipment Problem Format

$$\text{Minimize } Z = 146x_{1,2} + 324x_{1,4} + 286x_{1,5} + \dots + 587x_{12,9} + 340x_{12,10} + 457x_{12,11}$$

s.t.

$$\begin{aligned} x_{1,2} + x_{1,4} + \dots + x_{1,12} - x_{2,1} - \dots - x_{12,1} &= 75 && \text{Supply} \\ x_{2,1} + x_{2,4} + \dots + x_{2,12} - x_{1,2} - \dots - x_{12,2} &= 125 && \text{Node} \\ x_{3,4} + x_{3,6} + \dots + x_{3,12} - x_{4,3} - \dots - x_{12,3} &= 100 && \text{Constraints} \end{aligned}$$

$$\text{[quantity shipped out]} - \text{[quantity shipped in]} = \text{original supply}$$

$$\begin{aligned} -x_{1,4} - x_{2,4} - \dots - x_{11,4} + x_{4,1} + x_{4,2} \dots + x_{4,11} &= 0 && \text{Junction} \\ -x_{1,5} - x_{2,5} - \dots - x_{12,5} + x_{5,1} + x_{5,2} \dots + x_{5,12} &= 0 && \text{Node} \\ -x_{2,6} - x_{3,6} - \dots - x_{12,6} + x_{6,2} + x_{6,3} \dots + x_{6,12} &= 0 && \text{Constraints} \\ -x_{2,7} - x_{3,7} - \dots - x_{12,7} + x_{7,2} + x_{7,3} \dots + x_{7,12} &= 0 && \\ -x_{3,8} - x_{5,8} - \dots - x_{12,8} + x_{8,3} + x_{8,5} \dots + x_{8,12} &= 0 && \end{aligned}$$

$$- \text{[quantity shipped in]} + \text{[quantity shipped out]} = 0$$

$$\begin{aligned} x_{1,9} + x_{2,9} + \dots + x_{12,9} - x_{9,1} - x_{9,2} \dots - x_{9,12} &= 80 && \text{Destination} \\ x_{1,10} + x_{2,10} + \dots + x_{12,10} - x_{10,1} - x_{10,2} \dots - x_{10,12} &= 65 && \text{Node} \\ x_{2,11} + x_{3,11} + \dots + x_{12,11} - x_{11,2} - x_{11,3} \dots - x_{11,12} &= 70 && \text{Constraints} \\ x_{1,12} + x_{2,12} + \dots + x_{11,12} - x_{12,1} - x_{12,2} \dots - x_{12,11} &= 85 && \end{aligned}$$

$$\text{[quantity shipped in]} - \text{[quantity shipped out]} = \text{original demand}$$

and $x_{ij} \geq 0$

The P&T Co. Transshipment Problem

LP Format 2: Transportation Problem Format

$$\text{Minimize } Z = 146x_{1,2} + 5000x_{1,3} + 324x_{1,4} + \dots + 587x_{12,9} + 340x_{12,10} + 457x_{12,11}$$

s.t.

$x_{1,1} + x_{1,2} + \dots + x_{1,11} + x_{1,12}$	$= 375$	<i>Supply Node Constraints</i>
$x_{2,1} + x_{2,2} + \dots + x_{2,11} + x_{2,12}$	$= 425$	
$x_{3,1} + x_{3,2} + \dots + x_{3,11} + x_{3,12}$	$= 400$	
$x_{4,1} + x_{4,2} + \dots + x_{4,11} + x_{4,12}$	$= 300$	
$x_{5,1} + x_{5,2} + \dots + x_{5,11} + x_{5,12}$	$= 300$	
$x_{6,1} + x_{6,2} + \dots + x_{6,11} + x_{6,12}$	$= 300$	
$x_{7,1} + x_{7,2} + \dots + x_{7,11} + x_{7,12}$	$= 300$	
$x_{8,1} + x_{8,2} + \dots + x_{8,11} + x_{8,12}$	$= 300$	
$x_{9,1} + x_{9,2} + \dots + x_{9,11} + x_{9,12}$	$= 300$	
$x_{10,1} + x_{10,2} + \dots + x_{10,11} + x_{10,12}$	$= 300$	
$x_{11,1} + x_{11,2} + \dots + x_{11,11} + x_{11,12}$	$= 300$	
$x_{12,1} + x_{12,2} + \dots + x_{12,11} + x_{12,12}$	$= 300$	

The P&T Co. Transshipment Problem

LP Format 2: Transportation Problem Format

$$\begin{array}{rcll} X_{1,1} + X_{2,1} + \dots + X_{11,1} + X_{12,1} & = & 300 & \text{Demand} \\ X_{1,2} + X_{2,2} + \dots + X_{11,2} + X_{12,2} & = & 300 & \text{Node} \\ X_{1,3} + X_{2,3} + \dots + X_{11,3} + X_{12,3} & = & 300 & \text{Constraints} \\ X_{1,4} + X_{2,4} + \dots + X_{11,4} + X_{12,4} & = & 300 & \\ X_{1,5} + X_{2,5} + \dots + X_{11,5} + X_{12,5} & = & 300 & \\ X_{1,6} + X_{2,6} + \dots + X_{11,6} + X_{12,6} & = & 300 & \\ X_{1,7} + X_{2,7} + \dots + X_{11,7} + X_{12,7} & = & 300 & \\ X_{1,8} + X_{2,8} + \dots + X_{11,8} + X_{12,8} & = & 300 & \\ X_{1,9} + X_{2,9} + \dots + X_{11,9} + X_{12,9} & = & 380 & \\ X_{1,10} + X_{2,10} + \dots + X_{11,10} + X_{12,10} & = & 365 & \\ X_{1,11} + X_{2,11} + \dots + X_{11,11} + X_{12,11} & = & 370 & \\ X_{1,12} + X_{2,12} + \dots + X_{11,12} + X_{12,12} & = & 385 & \end{array}$$

and

$$x_{ij} \geq 0$$

The P&T Co. Transshipment Problem

PROC TRANS Format: Transportation Problem Format

```
DATA NEW;
INPUT  SOURCE1-SOURCE3 TRNSHP4-TRNSHP8 DESTIN9-DESTIN12 SUPPLY SOURCE$;
CARDS;
  300    300    300    300    300    300    300    300    380    365    370    385    .    .
    0    146  5000    324    286  5000  5000  5000    452    505  5000    871  375  SOURCE1
  146     0  5000    373    212    570    609  5000    335    407    688    784  425  SOURCE2
5000  5000     0    658  5000    405    419    158  5000    685    359    673  400  SOURCE3
  322    371    656     0    262    398    430  5000    503    234    329  5000  300  TRNSHP4
  284    210  5000    262     0    406    421    644    305    207    464    558  300  TRNSHP5
5000  569    403    398    406     0     81    272    597    253    171    282  300  TRNSHP6
5000  608    418    431    422     81     0    287    613    280    236    229  300  TRNSHP7
5000  5000    158  5000    647    274    288     0    831    501    293    482  300  TRNSHP8
  453    336  5000    505    307    599    615    831     0    359    706    587  300  DESTIN9
  505    407    683    235    208    254    281    500    357     0    362    341  300  DESTIN10
5000  687    357    329    464    171    236    290    705    362     0    457  300  DESTIN11
  868    781    670  5000    558    282    229    480    587    340    457     0  300  DESTIN12
;
PROC TRANS  ;
  TAILNODE  SOURCE;
  HEADNODE  SOURCE1-SOURCE3 TRNSHP4-TRNSHP8 DESTIN9-DESTIN12;
  SUPPLY    SUPPLY;
  TITLE1   'H&L TRANSSHIPMENT PROBLEM';
  TITLE2   'OPTIMAL Z = 145175';
RUN;
PROC PRINT;
RUN;
```

The P&T Co. Transshipment Problem Solution

H&L TRANSSHIPMENT PROBLEM
OPTIMAL Z = 145175

			S	S	S	T	T	T	T	T	D	D	D		
	S	S	O	O	O	R	R	R	R	R	E	S	S	S	—
	O	U	U	U	U	N	N	N	N	N	S	T	T	T	D
	U	P	R	R	R	S	S	S	S	S	T	I	I	I	U
0	R	P	C	C	C	H	H	H	H	H	I	N	N	N	A
B	C	L	E	E	E	P	P	P	P	P	N	1	1	1	L
S	E	Y	1	2	3	4	5	6	7	8	9	0	1	2	—
1	_DEMAND_	.	300	300	300	300	300	300	300	300	380	365	370	385	.
2	SOURCE1	375	300	0	0	0	75	0	0	0	0	0	0	0	0
3	SOURCE2	425	0	300	0	0	0	0	0	0	80	45	0	0	-86
4	SOURCE3	400	0	0	300	0	0	0	0	30	0	0	70	0	-194
5	TRNSHP4	300	0	0	0	300	0	0	0	0	0	0	0	0	-324
6	TRNSHP5	300	0	0	0	0	225	0	0	0	0	75	0	0	-286
7	TRNSHP6	300	0	0	0	0	0	300	0	0	0	0	0	0	-599
8	TRNSHP7	300	0	0	0	0	0	0	300	0	0	0	0	0	-613
9	TRNSHP8	300	0	0	0	0	0	0	0	270	0	0	0	30	-352
10	DESTIN9	300	0	0	0	0	0	0	0	0	300	0	0	0	-421
11	DESTIN10	300	0	0	0	0	0	0	0	0	0	245	0	55	-493
12	DESTIN11	300	0	0	0	0	0	0	0	0	0	0	300	0	-553
13	DESTIN12	300	0	0	0	0	0	0	0	0	0	0	0	300	-834
14	_DUAL_	.	0	86	194	324	286	599	613	352	421	493	553	834	.

LP Format 1 **p**

Obj = 145,175

X1,5 = 75 X2,9 = 80 X2,10 = 45 X3,8 = 30 X3,11 = 70 X5,10 = 75 X8,12 = 30 X10,12 = 55

LP Format 2 **p**

Obj = 145,175

X1,1 = 300 X1,5 = 75 X2,2 = 300 X2,9 = 80 X2,10 = 45 X3,3 = 300 X3,8 = 30 X3,11 = 70
 X4,4 = 300 X5,5 = 225 X5,10 = 75 X6,6 = 300 X7,7 = 300 X8,8 = 270 X8,12 = 30 X9,9 = 300
 X10,10 = 245 X10,12 = 55 X11,11 = 300 X12,12 = 300