Stochastic Programming

[Hillier & Lieberman, *Intro. to Math. Programming*, p. 364-368]

One-Stage Problem:

Assume a_{ij} and b_i are random variables. Each of the a_{ij}'s and b_i's with multiple possible values would be replaced by its most restrictive value for the constraint:

$$3\ (max\ a_{ij})\ x_j \leq min\ b_i$$

Example 1:

$$a_{11}x1 + a_{12}x2 \leq b_1$$

where a_{11}, a_{12}, b_1 are random variables having the following possible values:

$1 \leq a_{11} \leq 2$ $\quad$ $2 \leq a_{12} \leq 3$ $\quad$ $4 \leq b_1 \leq 5$

Reformulating the constraint in its most restrictive form yields:

$$2x1 + 3x2 \leq 4$$

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One-Stage Problem:

<u>Example 2</u>:

$$a_{11}x1 + a_{12}x2 \leq b_1$$

where a_{11}, a_{12}, b_1 are random variables having the following possible values:

a_{11} = 1 or 2 a_{12} = 2 or 3 b_1 = 4 or 5

Further suppose that only two scenarios are possible:

Scenario 1:	$a_{11} = 1$	$a_{12} = 3$	$b_1 = 4$
Scenario 2:	$a_{11} = 2$	$a_{12} = 2$	$b_1 = 5$

Reformulating the constraint in its most restrictive form yields:

$$x1 + 3x2 \leq 4$$
$$2x1 + 2x2 \leq 5$$

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Multistage Problem:

Example:

Maximize $Z = 3x1 + 7x2 + 11x3$

s.t. $a_{11}x1 + a_{12}x2 + a_{13}x3 \leq 100$

and $x1 \geq 0 \quad x2 \geq 0 \quad x3 \geq 0$

where

$a_{11} =$ 1 with a probability of (½)
2 with a probability of (½)

$a_{12} =$ 3 with a probability of (½)
4 with a probability of (½)

$a_{13} =$ 5 with a probability of (½)
6 with a probability of (½)

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Multistage Problem:

Replace *x2* by a new set of decision variables:

x21 = value chosen for *x2* if $a_{11} = 1$
x22 = value chosen for *x2* if $a_{11} = 2$

And replace *x3* by a new set of decision variables:

x31 = value chosen for *x3* if $a_{11} = 1$ & $a_{12} = 3$
x32 = value chosen for *x3* if $a_{11} = 1$ & $a_{12} = 4$
x33 = value chosen for *x3* if $a_{11} = 2$ & $a_{12} = 3$
x34 = value chosen for *x3* if $a_{11} = 2$ & $a_{12} = 4$

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Multistage Problem:

The reformulated problem is:

$$\text{Max } E(Z) = 3x1 + 7(½)\, x21 + 7(½)\, x22 + 11(1/4)\, x31 + 11(1/4)\, x32 + 11(1/4)\, x33 + 11(1/4)\, x34$$

or

$$\text{Max } E(Z) = 3x1 + 7(½)\,(x21 + x22) + 11(1/4)\,(x31 + x32 + x33 + x34)$$

s.t.

$$x1 + 3x21 + 6x31 \leq 100$$
$$x1 + 4x21 + 6x32 \leq 100$$
$$2x1 + 3x22 + 6x33 \leq 100$$
$$2x1 + 4x22 + 6x34 \leq 100$$