Simulations with Continuous Random Variables

The basic procedure for generating continuous random variables is to first generate a uniform (0,1) random number and then transform it into a random variable from a specified distribution.

Inverse Transformation Method (ITM)

The inverse transformation method is generally used for distributions whose cumulative distribution function can be obtained in closed form (exponential, uniform, and triangular distributions).

Step 1: Given a probability density function f(x), obtain the cumulative distribution function F(x)

Step 2: Generate a uniform (0,1) random number, r

Step 3: Set F(x) = r and solve for x [*transform r into x*]

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Example -- The Uniform Distribution

probability density function	***cumulative distribution function***
$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$	$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$

Using the ITM, $\frac{x - a}{b - a} = r$

Solving for x yields, $x = a + (b - a)\, r$

SAS : **x = a + (b - a) * RANUNI(seed)**

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Example -- The Triangular Distribution

The triangular distribution is frequently used to represent non-normal distributions. It has important implications in simulation and is often used to represent activities for which there are few or no data.

It is defined by just 3 parameters:

a = minimum value b = maximum value m (or c) = mode

Mean : $E[x] = (a + m + b) /3$

Variance : $V[x] = [(b - a)^2 + (m - a)(m - b)] / 18$

<u>Probability</u> <u>density function</u> is defined as:

$f_1(x) = 2(x-a) / [(b-a)(m-a)]$ for $a \leq x \leq m$

$f_2(x) = 2(b-x) / [(b-a)(b-m)]$ for $m \leq x \leq b$

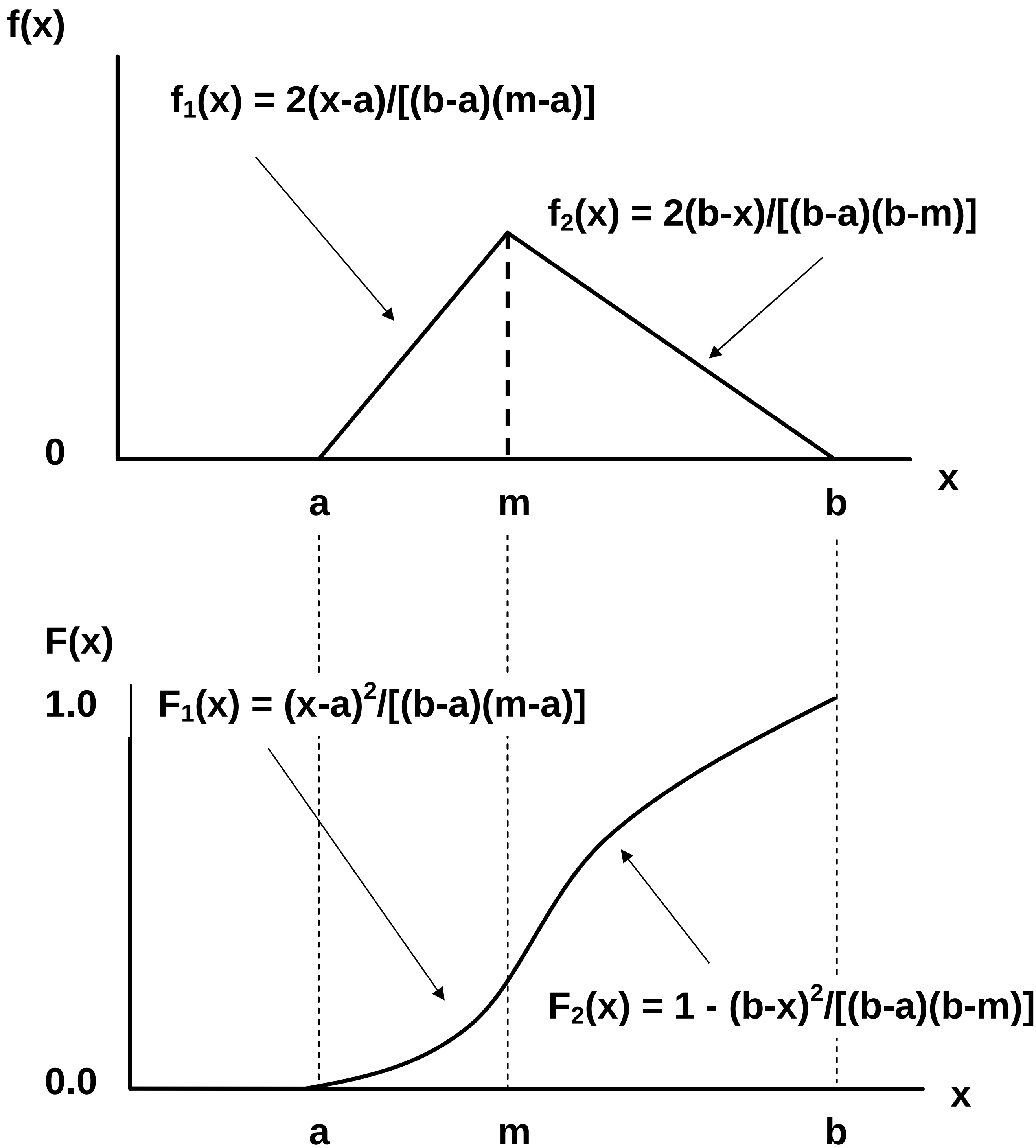
<u>Cumulative</u> <u>distribution function</u> is defined as:

$F_1(x) = (x-a)^2 / [(b-a)(m-a)]$ for $a \leq x \leq m$

$F_2(x) = 1 - (b-x)^2 / [(b-a)(b-m)]$ for $m \leq x \leq b$

SAS : **x = (b - a) * RANTRI (seed, (m - a) / (b - a)) + a**

PDF and CDF for a Triangular Distribution



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Example – *The Normal Distribution*
-- The Direct Method (Box-Muller Transformation)

The Direct Method, developed by Box and Muller (1958), generates 2 uniform (0,1) random numbers, r_1 and r_2 and then transforms them into two standardized normal random variables with mean=0 and variance=1.

$$\mathbf{Z_1} = (-2 \ln r_1)^{1/2} \sin 2 \, pi \, r_2$$
$$\mathbf{Z_2} = (-2 \ln r_1)^{1/2} \cos 2 \, pi \, r_2$$

These standard normal variables are then transformed into normal variables by:

$$\mathbf{X_1} = \text{mean} + \text{stddev}\ \mathbf{Z_1}$$
$$\mathbf{X_2} = \text{mean} + \text{stddev}\ \mathbf{Z_2}$$

SAS : **x = mu + sqrt(sigmasq) * RANNOR (seed)**

or

x = mu + sqrt(sigmasq) * NORMAL (seed)